Homework 1

Integer and Floating Point Number Representations

Integer

Problem 1

Suppose you have a 1 GHz x64 core and you can execute three integer operations (additions or subtractions) every cycle. How long (how many seconds) will the following loop run?

```
long i; /* 64 bit signed integer */
long s;
for (i = 0 ; i >= 0; i++) {
    s += i;
}
```

Problem 2

What logical operator is equivalent to multiplying two single-bit integers together? How would you multiply two two-bit integers together using logic and addition? How would you generalize to multiplying two n-bit integers?

Problem 3

Some instruction sets, including x64, provide an integer representation in addition to two's complement. This representation is called Binary Coded Decimal (BCD). In BCD, a decimal digit (0,1,2,3,4,5,6,7,8,9) is encoded into a group of 4 bits using 0000 through 1001. How many unique numbers can be represented in a 64 bit BCD quantity? Why might one use BCD to represent prices like \$10.99?

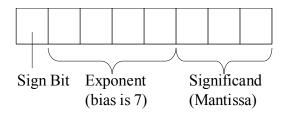
Problem 4

Many instruction sets have instructions where when you multiply two k bit numbers the result is stored as two k bit numbers. Why? Similarly, many have instructions where if you divide two k bit numbers, the result is stored as two k bit numbers. Why?

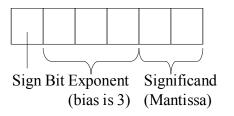
Floating Point

Consider the following two small floating point formats based on the IEEE standard:

• Little Format



• Tiny Format



Except for the sizes of these formats, the rules are those of the IEEE standard.

Problem 1

For both formats, determine the following values (in decimal)

- 1. Largest positive finite number
- 2. Positive normalized number closest to zero
- 3. Largest negative denormalized number
- 4. Negative denormalized number closest to zero

Problem 2

Encode the following values in the 8 bit Little Format: $\frac{3}{4}$, -13/16, 44, -104, NaN, and negative infinity. Show each in binary and hexadecimal.

Problem 3

Determine the values corresponding to the following Little Format bit patterns. The leftmost bit is the most significant

- 1. 10101011
- 2. 011111000
- 3. 10110101
- 4. 01011111
- 5. 11000101
- 6. 11111111

Problem 4

Convert the following 8 bit Little Format numbers into 6 bit Tiny Format numbers. Overflow should yield +/- infinity, underflow should yield +/- 0.0, and rounding should follow the "round-to-nearest-even" tie-breaking rule. Show the bit pattern and its hex representation.

- 1. 00010010
- 2. 11101011
 3. 10100011
- 4. 11001110
- 5. 00110101 6. 11111111
- 7. 01111000